Primordial black holes from the inflating curvaton

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The primordial black hole (PBH) formation is studied in the light of the inflating curvaton. The typical scale of the PBH formation is determined by curvaton inflation, which may generate PBH with $10^{14}\mathrm{g} \leq M_{\mathrm{PBH}} \leq 10^{38}\mathrm{g}$ when curvaton inflation gives the number of e-foldings $5 \leq N_2 \leq 38$. The non-Gaussianity of the inflating curvaton does not prevent the PBH formation.

I. INTRODUCTION

The origin of the structure in the Universe is due to the primordial density perturbations that exist already when cosmological scales start entering the horizon after the final reheating.

Inflation can generate the primary source of the cosmological perturbation, which is usually the vacuum fluctuation of a light scalar field. In the original inflation model the primary perturbation of the light field (i.e, the inflaton perturbation) is converted into the curvature perturbation at the same time when the perturbation leaves the horizon.[1]

However, later studies revealed that there are many other candidates for the mechanism of generating the curvature perturbations after inflation [2–9, 11, 12]. The idea of these alternatives is very simple; given that there are many scalar fields in the particle physics model, there will be many fields displaced from their minima at the end of the primordial inflation epoch, which leads to the multi-component Universe after inflation where any kind of isocurvature perturbations can be created. Then the isocurvature perturbations can source the creation of the curvature perturbations after inflation.

In this paper we consider inflating curvaton [9–13] that can lead to the PBH formation. Generation of the cosmic microwave background (CMB) perturbations is separated from the PBH formation, since unification of these perturbations requires highly model-dependent argument that is not suitable for the purpose of this paper. Previously, such unification scenario has been considered for the running-mass model or the Type-III Hilltop inflation model [14–18]. PBH can also be produced from passive fluctuations [19]. (See also Ref.[20].) Our strategy is to separate PBH formation from the original inflation scenario, so that PBH formation is liberated from the CMB conditions. Note however that the seed perturbations are generated during primordial inflation.

Although not mandatory, the secondary inflation may solve cosmological problems of some specific models. For instance:

1. The primordial inflation model may require e-foldings less than 60 so that the model can explain the observed CMB. The short inflation could be needed to explain the spectral index and its runnning, or the inflation could be short because of the fast (rapid) rolling.

2. The model may predict unwanted relics after reheating. They must be diluted.

3. The model may not have elementary dark matter (DM) candidate. Then the dark matter of the Universe must be explained by something other than the particles.

The most optimistic scenario of the inflating curvaton is that all these "problems" are solved by the secondary curvaton inflation.

In the standard curvaton mechanism, the crucial assumption is that there are two components in the Universe, where one component scales like matter ($\rho_\sigma \propto a^{-3}$) while the other is the radiation ($\rho_r \propto a^{-4}$). The inflating curvaton uses slow-roll (or sometimes fast-roll [21]) field instead of the oscillating one, which is a natural generalization of the usual curvaton mechanism. We consider the slow or fast-rolling curvaton (the inflating curvaton that scales like $\rho_\sigma \propto a^{-3\epsilon_w}$) in the presence of the radiation. Then the curvature perturbation is expressed as [2, 6]

$$\zeta = (1-f)\zeta_r + f\zeta_\sigma, \tag{1}$$

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[1] This mechanism has been used for the PBH formation in Ref. [1], where the secondary inflation is considered for the creation of the perturbation. This must be distinguished from the curvaton mechanism.

where

$$f \equiv \frac{\dot{\rho}_\sigma}{\dot{\rho}_r + \dot{\rho}_\sigma} = \frac{3\epsilon_w \rho_\sigma}{3\epsilon_w \rho_\sigma + 4\rho_r}$$

$$\zeta_r \equiv -H\frac{\delta\rho_r}{\dot{\rho}_r} = \frac{\delta\rho_r}{4\rho_r}$$

$$\zeta_\sigma \equiv -H\frac{\delta\rho_\sigma}{\dot{\rho}_\sigma} = \frac{1}{3\epsilon_w}\frac{\delta\rho_\sigma}{\rho_\sigma}. \qquad (2)$$

Here ϵ_w is defined as

$$\dot{\rho}_\sigma = -3\epsilon_w H\rho_\sigma. \qquad (3)$$

Unlike the usual curvaton mechanism, the constancy of ζ_σ requires some debate in the practical calculation, since there is no doubt that ζ_σ is time-dependent when the radiation density is comparable to the curvaton density. However, after the beginning of the curvaton inflation, the ratio of the radiation decreases rapidly, and ϵ_w will soon behave like constant. In that sense the initial quantity of ζ_σ must be calculated in the inflationary phase slightly after a few e-foldings, which however must not be later than the significant evolution of the curvature perturbation [9, 12]. In this paper $t = t_{\text{ini}}$ is the time when the initial quantities are defined. Note that in the inflating curvaton model significant evolution occurs when $\epsilon_w \ll 1$, while in the usual curvaton it occurs when $\epsilon_w = 1$. The significant evolution is accompanied by the significant change of the parameter r_σ, which will be defined later. The curvature perturbation cannot evolve after $r_\sigma \simeq 1$.

In this paper we are assuming that the effective curvaton potential is always quadratic $(= \pm\frac{1}{2}m_\sigma^2\sigma^2)$ so that the ratio $\delta\sigma/\sigma$ behaves like constant after the horizon exit. Thinking about the scale dependence of the perturbation, "the initial $\delta\sigma/\sigma$ measured at the horizon exit" depends on the corresponding scale if σ is moving during the primordial inflation, and finally the distribution of $\delta\sigma/\sigma$ becomes scale-dependent. For the curvaton mechanism, small σ at the horizon exit means large perturbation $\propto \delta\sigma/\sigma$ on that scale.[2] If the effective curvaton mass is determined by the main component of the Universe, one can expect small mass during the radiation dominated epoch [22]. With these simple assumptions we examine the PBH formation in the generalized curvaton mechanism.

II. SCALE-DEPENDENT PERTURBATION FOR THE INFLATING CURVATON

To show an illustrative scenario we consider a new inflation scenario or a Type-I Hilltop inflation [14] for the

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[2] When we were finalizing the manuscript we found a new paper [23], in which the usual curvaton mechanism has been considered for a similar PBH formation. The secondary inflation is the crucial difference, since it expands the PBH scale and usually makes PBH heavier than the usual curvaton scenario.
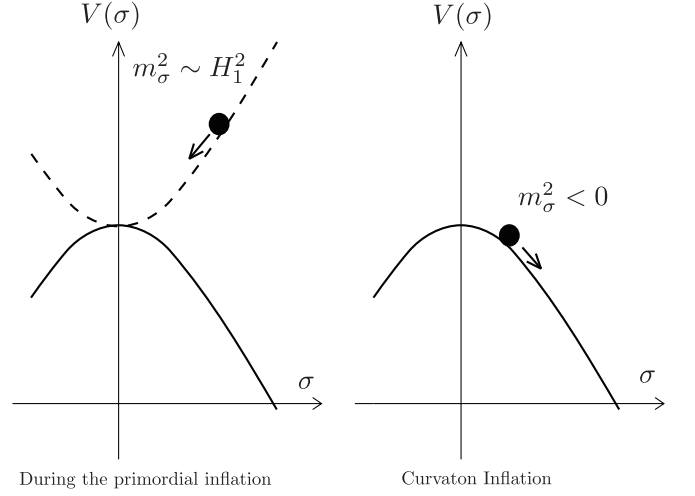
FIG. 1: The curvaton potential is flipped during the primordial inflation because of the Hubble-induced mass, while it is negative during the secondary (curvaton) inflation. The density of the curvaton is subdominant during the primordial inflation.

inflating curvaton, in which the curvaton potential satisfies $\epsilon_\sigma \equiv \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2 \ll 1$ during the curvaton inflation [9, 24].

Consider the effective potential for the inflating curvaton;

$$V(\sigma) = V_0 + \frac{1}{2}m_\sigma(H)^2\sigma^2, \qquad (4)$$

where $m_\sigma^2 < 0$ is possible. Here the effective mass is determined by the main component of the Universe. As is shown in Fig.1, we are assuming $m_\sigma^2 > 0$ during the primordial inflation while $m_\sigma^2 < 0$ during the curvaton inflation. Hereafter we use the index "1" for the primordial inflation and "2" for the curvaton inflation, respectively.

In this paper we are assuming quadratic potential for the curvaton so that σ and $\delta\sigma$ can share the identical equation of motion. At the same time the equation becomes a linear differential equation and consequently the ratio $\delta\sigma/\sigma$ behaves like constant. We are avoiding any deviation from the quadratic assumption.

Let us see why the ratio can bahave like constant when the quadratic potential is assumed for the curvaton. For the simplest example, consider the equations when m_σ and H are constants:

$$\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + m_\sigma^2(H)\sigma = 0, \qquad (5)$$

where the equation has the solution of the form $\sigma \propto e^{-\alpha t}$, and the coefficient α is given by

$$\alpha \equiv \frac{3}{2}H\left[1 \pm \sqrt{1 - \left(\frac{2m_\sigma}{3H}\right)^2}\right]. \qquad (6)$$

Defining $\beta \equiv \alpha/H$, we find

$$\sigma(t) \sim \sigma(0)e^{-\beta H t} \equiv \sigma(0)e^{-\beta \Delta N}, \qquad (7)$$

where $\Delta N \equiv Ht$ is the number of e-foldings spent after the horizon exit.[3] For the curvaton perturbation ($\sigma \equiv \bar{\sigma} + \delta\sigma$), the equation of motion after the horizon exit gives [21] $\delta\sigma_k(t) \sim \delta\sigma_k(0)e^{-\beta \Delta N}$. We thus find $\delta\sigma_k(t)/\sigma(t) \simeq \delta\sigma_k(0)/\sigma(0)$ because of the cancellation of the evolution factor. Although the mass flips its sign during the evolution, one can take the initial condition everytime at the beginning of the epoch to find similar evolution factor. Here the growing solution ($\beta < 0$) is possible when the mass term has the negative sign [24]. The equation of motion gives either fast or slow-rolling solution when $|\eta_\sigma| < 3/4$ (i.e, when $|m_\sigma| < \frac{3}{2}H$), where the eta-parameter is defined by $\eta_\sigma \equiv \frac{m_\sigma^2}{3H^2}$. As a consequence of the cancellation, the ratio $\delta\sigma/\sigma$ behaves like constant as far as the evolution is given by the separable function. (See also Sec.II A.)

For the inflating curvaton model, the source of the curvature perturbation is $\delta\sigma$, which leaves horizon during the primordial inflation and evolves thereafter until the curvaton mechanism starts to work. At the horizon exit, the spectrum is given by

$$\mathcal{P}_{\delta\sigma_*} = \left(\frac{H_1}{2\pi}\right)^2, \qquad (8)$$

where H_1 is the Hubble parameter during the primordial inflation.

Since we have two inflation stages (primordial and curvaton), we also need to consider horizon exit during the curvaton inflation. Indeed, Ref.[1] uses the perturbation that leaves horizon during the second inflation and explains the small-scale perturbations.[4] We are considering the opposite scenario, in which the perturbation that exits horizon during the primordial inflation seeds the curvaton mechanism. The scale dependence of the perturbation is illustrated in Fig.2. We are focusing on the specific case in which the amplitude of the curvaton perturbation becomes strongly scale dependent due to the Hubble-induced mass during the primordial inflation. For that reason we are not avoiding the η-problem of the curvaton potential.

In our scenario the seed perturbation (the curvaton perturbation) created during primordial inflation is

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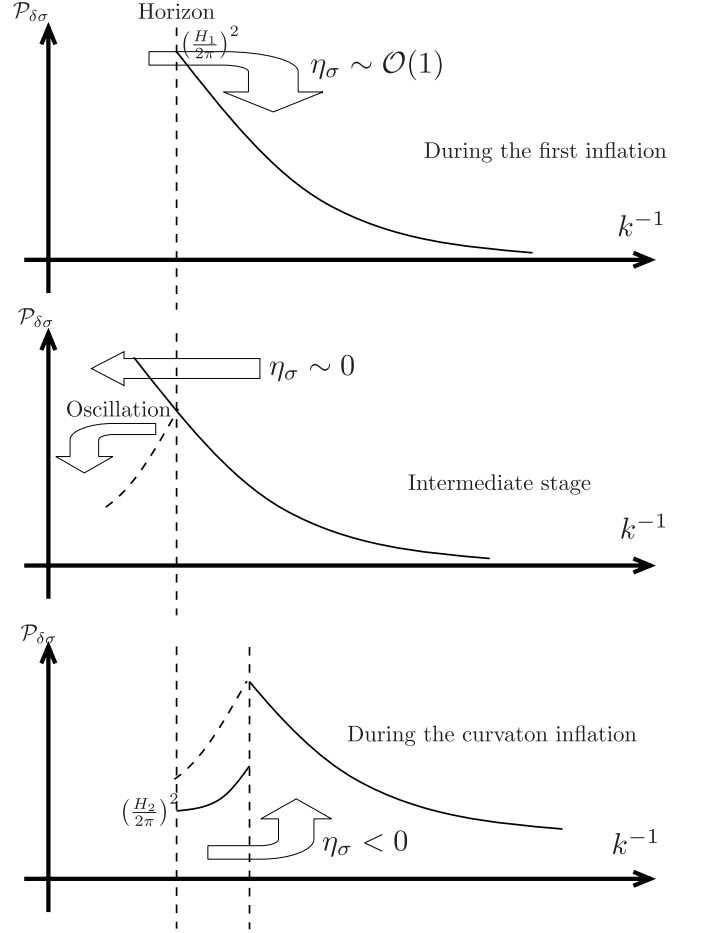[3] Here we have omitted the subscripts, but the evolution must be calculated for both inflationary stages.

[4] Another scenario has been considered in Ref.[14–18]. Given that the curvature perturbation at the scale is given by

$$\mathcal{P}_{\zeta_\varphi} \simeq \frac{\mathcal{P}_{\delta\varphi}}{2\epsilon_\varphi M_p^2}, \qquad (9)$$

the significant scale-dependence may appear either from $\epsilon_\varphi(k)$ (running potential) or from $\mathcal{P}_{\delta\varphi}(k)$. Our paper complements the approach given in Ref.[17] for the running $\epsilon_\varphi(k)$.

FIG. 2: The field perturbation has significant scale dependence. The peak appears at k_{PBH}^{-1}, which corresponds to the scale that touches the horizon at the beginning of the curvaton inflation. In this picture the amplitude of the perturbation that leaves horizon during the first inflation decreases during primordial inflation ($\mathcal{T}_1 \ll 1$), while it increases during curvaton inflation ($\mathcal{T}_2 \geq 1$). The perturbation entering into the horizon during the intermediate epoch causes oscillation around its average and decreases its amplitude. In the third picture, the perturbation newly created during the curvaton inflation is not dominating the amplitude at k_{PBH}.

highly scale-dependent and it is converted into the curvature perturbation during the curvaton inflation. Our scenario complements the running-mass model or the Type-III Hilltop inflation model considered in Ref. [14–18].

For later use, we define the evolution of the perturbations as

$$\mathcal{P}_{\delta\sigma}^{1/2}(t_{\mathrm{ini}}) \simeq \frac{H_I}{2\pi} \times \mathcal{T}_1 \mathcal{T}_{int} \mathcal{T}_2, \qquad (10)$$

where t_{ini} denotes the time when the inflating curvaton starts to work. Here the k-dependent functions $\mathcal{T}_1$, $\mathcal{T}_{int}$ and $\mathcal{T}_2$ denote the translation functions during primordial inflation ($\mathcal{T}_1 \sim e^{-\beta_1 \Delta N_1} \ll 1$ for $\beta_1 > 0$), intermediate evolution ($\mathcal{T}_{\mathrm{int}} \simeq 1$ for $\beta_{\mathrm{int}} \sim 0$) and the second inflation ($\mathcal{T}_2 \sim e^{-\beta_2 \Delta N_2} > 1$ for $\beta_2 < 0$). Here ΔN_1

and ΔN_2 are the number of e-foldings elapsed during each inflation stage, defined for the perturbation on that scale ($k = k_{\mathrm{PBH}}$). ΔN_1 is defined after the perturbation leaves horizon, and ΔN_2 is defined before inflating curvaton starts to work at $t = t_{\mathrm{ini}}$. $\mathcal{T}_2$ is usually negligible for the inflating curvaton.

Here the inhomogeneity ($\delta\sigma$) entering into the horizon can never be kept frozen even if the potential is flat. Oscillation around the average (not around the true vacuum) starts just after the horizon entry and the amplitude is decreasing in the expanding Universe. If the oscillation is sinusoidal, the evolution of the amplitude can be approximated as $\propto a^{-3/2}$, which causes significant reduction of the amplitude, as is illustrated in the second picture of Fig.2. We thus find a peak on the scale that touches the horizon at the beginning of the curvaton inflation. We are using this scale for the PBH formation.

For the PBH formation, we consider the spectrum of the source perturbation ($\delta\sigma/\sigma$), focusing on the scale corresponding to the peak. The scale of the PBH satisfies

$$
\begin{aligned}
\ln\left(\frac{k_{\mathrm{PBH}}}{a_0 H_0}\right) \sim\ & 62 - \Delta N_1 - N_2 - \frac{1}{2}\ln\left(\frac{10^{14}GeV}{H_1}\right) \\
& -\frac{1}{12}\ln\left(\frac{3H_1^2 M_p^2}{\rho_{R1}}\right) \\
& -\frac{1}{12}\ln\left(\frac{3H_2^2 M_p^2}{\rho_{R2}}\right).
\end{aligned} \tag{11}
$$

Here H_i is the Hubble parameter during inflation, and ρ_{Ri} is the energy density at the reheating after each inflation stage. The reheating after the first inflation could be avoided, but for simplicity we are assuming the reheating.

Since the perturbation ($k = k_{\mathrm{PBH}}$) does not enter the horizon before the curvaton inflation, there is a bound for ΔN_1;[5]

$$
\begin{aligned}
\Delta N_1 \geq\ & \frac{1}{6}\ln\left(\frac{3H_1^2 M_p^2}{\rho_{R1}}\right) + \frac{1}{4}\ln\left(\frac{\rho_{R1}}{3H_2^2 M_p^2}\right) \\
\simeq\ & 9 + \frac{2}{3}\ln\left(\frac{M_1}{10^{15}\mathrm{GeV}}\right) - \ln\left(\frac{M_2}{10^{10}\mathrm{GeV}}\right) \\
& + \frac{1}{3}\ln\left(\frac{T_{R1}}{10^{12}\mathrm{GeV}}\right),
\end{aligned} \tag{12}
$$

where the scales are defined by $M_1^4 \equiv 3H_1^2 M_p^2$ and $M_2 \equiv 3H_2^2 M_p^2$.

Although the evolution of $\delta\sigma$ is highly non-trivial, our final result does not depend explicitly on the evolution after horizon exit, as far as the curvaton potential is

quadratic. That is because the ratio $\delta\sigma/\sigma$ behaves like constant after the horizon exit and thus the quantity evaluated just at the horizon exit is what we need for the PBH formation. Therefore, we find the perturbation using $\delta\sigma/\sigma$, which is evaluated at the horizon exit [9, 12]:

$$
\begin{aligned}
\zeta \simeq\ & r_\sigma \frac{2R}{3\epsilon_w}\frac{\delta\sigma}{\sigma} \\
\simeq\ & \frac{r_\sigma A}{3\eta_2}\frac{\delta\sigma}{\sigma},
\end{aligned} \tag{13}
$$

where the equation of motion is approximated by $AH\dot\sigma \simeq -\partial V/\partial\sigma$ with the coefficient $A = \beta$ for the fast-rolling and $A = 3$ for the slow-rolling. Here η_2 denotes the value of η_σ during the curvaton inflation. The fraction

$$
\begin{aligned}
r_\sigma \equiv\ & \frac{3\epsilon_w \rho_\sigma}{3\epsilon_w \rho_\sigma + 4\rho_R} \\
\simeq\ & \frac{3\epsilon_w}{3\epsilon_w + 4e^{-4N_2}}
\end{aligned} \tag{14}
$$

is defined at the end of curvaton inflation and throughout this paper we expect $r_\sigma \sim 1$ for simplicity. In the above calculation we have used the ratio defined by $R \equiv \frac{m_\sigma^2 \sigma^2}{2V_0}$ ($m_\sigma^2 < 0$ is assumed during the curvaton inflation) and the obvious relations

$$
\begin{aligned}
\dot\rho_\sigma \equiv\ & -3\epsilon_w H_2 \rho_\sigma \\
\dot\rho_\sigma \simeq\ & \frac{\partial V}{\partial\sigma}\dot\sigma.
\end{aligned} \tag{15}
$$

The PBH is formed when the perturbation with the significant density contrast ($\delta\rho/\rho \geq 0.1$) enters the horizon. In our case the PBH formation occurs when the perturbation of the scale $\sim k_{\mathrm{PBH}}$ becomes accessible within the horizon. The typical mass of the PBH is given by [18, 25]

$$
\frac{k_{\mathrm{PBH}}}{0.00974\mathrm{Mpc}^{-1}} \simeq \left(\frac{g_*}{g_{*eq}}\right)^{-\frac{1}{12}}\left(\frac{M_{\mathrm{PBH}}}{6.67\times 10^{50}\mathrm{g}}\right)^{-\frac{1}{2}}, \tag{16}
$$

which gives

$$
M_{\mathrm{PBH}} \simeq 10^{46}\mathrm{g}\left(\frac{g_*}{100}\right)^{-\frac{1}{6}}\left(\frac{k_{\mathrm{PBH}}}{1\mathrm{Mpc}^{-1}}\right)^{-2}, \tag{17}
$$

where g_* is the degrees of freedom in the radiation. Let us see more details of the scenario.

First, we need to check that ζ at the PBH scale is dominated by the curvaton. Since the creation of $\delta\sigma$ is possible also in the secondary inflation epoch, we need to compare it with the one generated during the primordial inflation.[6] We thus need the following condition at the

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[5] The number of e-foldings during inflation is usually given by $N = \ln\frac{a(t_e)H(t_e)}{a(t)H(t)} \simeq \ln\frac{a(t_e)}{a(t)}$. In the above calculation the factor $1/6$ appears because H varies.

[6] The spectrum of $\delta\sigma$, which is (newly) created during curvaton inflation, must not dominate the perturbation on the PBH scale. Since otherwise the curvaton mechanism is irrelevant for the PBH formation.

beginning of the secondary inflation;

$$\frac{H_2}{2\pi} < \frac{H_1}{2\pi} e^{-\beta_1 \Delta N_1}, \tag{18}$$

which gives a rather trivial condition

$$\frac{M_2}{M_1} < e^{-\frac{\beta_1}{2}\Delta N_1}. \tag{19}$$

Second, since the PBH formation requires significant density contrast, we need

$$\mathcal{P}_{\zeta_\sigma}^{1/2} \simeq \frac{r_\sigma A}{3\eta_2} \frac{H_1}{2\pi\sigma_{k_\mathrm{PBH}}} \sim 0.1, \tag{20}$$

where σ_{k_PBH} denotes the value of σ when the perturbation ($k = k_\mathrm{PBH}$) leaves the horizon. Then we find

$$\begin{aligned} \sigma_{k_\mathrm{PBH}} &\sim \left(\frac{r_\sigma A}{3\eta_2}\right) \frac{H_1}{2\pi} \\ &\sim 10^{11}\mathrm{GeV} \left(\frac{r_\sigma A}{3\eta_2}\right) \left(\frac{M_1}{10^{15}\mathrm{GeV}}\right)^2. \end{aligned} \tag{21}$$

Third, ζ at the CMB scale must not be dominated by the curvaton. The consistency at the CMB scale requires

$$\mathcal{P}_{\zeta_\sigma}^{1/2} \simeq \frac{r_\sigma A}{3\eta_2} \frac{H_1}{2\pi\sigma_{k_\mathrm{CMB}}} < 10^{-5}. \tag{22}$$

Therefore, from Eq.(20) and Eq.(22), we find

$$\begin{aligned} \frac{\sigma_{k_\mathrm{PBH}}}{\sigma_{k_\mathrm{CMB}}} &\simeq e^{-\beta_1(N_1 - \Delta N_1)} \\ &< 10^{-4}. \end{aligned} \tag{23}$$

This condition suggests that σ moves significantly during the primordial inflation stage. We thus find

$$N_1 - \Delta N_1 > 9.2 \times \frac{1}{\beta_1}. \tag{24}$$

Here the scale can be given by

$$\left(\frac{k_\mathrm{CMB}}{k_\mathrm{PBH}}\right) \simeq e^{N_1 - \Delta N_1}, \tag{25}$$

where $k_\mathrm{CMB} \le 1\mathrm{Mpc}^{-1}$. Note that steep potential allows significant running of the perturbation that leads to small k_{PBH} (heavy PBH).[7]

According to Carr et al in Ref [26], only PBHs with $10^{21}\mathrm{g} < M_\mathrm{PBH} < 10^{28}\mathrm{g}$ and $10^{35}\mathrm{g} < M_\mathrm{PBH} < 10^{36}\mathrm{g}$ can be allowed to become dark matter.[8] By observing μ-distortion due to the dissipation of large density

fluctuation at a small scale after the decoupling of the double-Compton scattering, we will be able to check the amplitude of small-scale fluctuation by PIXIE [28].

As an illuminating example, consider $M_1 \sim 10^{15}\mathrm{GeV}$, $M_2 \sim 10^{10}\mathrm{GeV}$ and $T_{Ri}/M_i \sim 10^{-3}$. These are giving a typical set of SUSY-GUT model. Then we find $N_1 - \Delta N_1 \simeq 10$ and $N_2 \sim 30$ (maximum curvaton inflation) gives $M_\mathrm{PBH} \simeq 10^{38}\mathrm{g}$, or $N_2 \sim 5$ (minimum curvaton inflation) gives $M_\mathrm{PBH} \simeq 10^{14}\mathrm{g}$. For $k_\mathrm{PBH} \sim 10^5\mathrm{Mpc}^{-1}$, we find $N_1 - \Delta N_1 \simeq 11.5$ and $\beta_1 > 0.8$, which allows $M_\mathrm{PBH} \sim 10^{36}\mathrm{g}$.

In the original argument of the inflating curvaton model, there was a prediction for the running spectral index (n') [9]. (Note that this "running" is not for the PBH perturbation. We are arguing here about the possible relation between the PBH mass and the running spectral index of the CMB.) A similar argument can be applied for the PBH model, which suggests that large N_2 (i.e, small N_1) gives both large M_PBH and enhanced running of the CMB spectrum.

To be more precise about the relation between the CMB spectrum and the PBH mass, let us assume that the primordial chaotic inflation is driven by the potential $\propto \phi^\alpha$ and that it generates CMB through the conventional curvaton mechanism. In that specific model we find the spectral index $n(k) - 1 \simeq -2\epsilon_1 = -\alpha/2N_1(k) \simeq -0.037 \pm 0.014$ with the running $n' \simeq -\alpha/2N_1^2 \simeq -(0.037\pm0.014)/N_1$. For the quadratic potential ($\alpha = 2$), one will find $N_1 \sim 30$ from the spectral index, which cannot be realized without the secondary inflation. The running of the spectral index is enhanced because of small N_1, which could be distinguished from other predictions. Note that in that model the second inflation is mandatory and the condition $N_1 + N_2 \simeq 60 - \ln(10^{-5}M_p/H_1)$ gives the PBH mass $M_\mathrm{PBH} \le 10^{38}g$. Here Eq.(12), which bounds ΔN_1 from below, is given by the scale of the inflating curvaton and thus it can be related to the theory beyond the standard model. For instance, the gravity mediated supersymmetry (SUSY) breaking may predict the scale of the inflating curvaton at $\sim 10^{10}\mathrm{GeV}$ and the inflating curvaton can dilute unwanted cosmological relics of the model such as gravitino or light moduli.

For the fast-roll inflation [21], the number of e-foldings is usually expected to be smaller than 60 ($N_1 < 60$). In that sense the secondary inflation is mandatory for the scenario. Also, inflating curvaton could have a very interesting application to the rapid roll inflation [29]. Using the inflating curvaton, one can increase the inflation scale of this model to an extent such that gravity waves could be generated without worrying about curvature perturbation. Anyway, the curvaton-like mechanism is inevitable if the primordial inflation may not generate required CMB spectrum. If the primordial inflation is short, the mechanism must be accompanied by the secondary inflation stage, which can be used either for CMB or for short-scale perturbations. In that sense the inflating curvaton is quite important for these models that do not have enough e-foldings by themselves.

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[7] Note however that Eq.(22) gives only the upper bound for the perturbation. The bound gives the heaviest PBH.

[8] CMB observations might have already exluded the latter parameter space. In addition, quite recently it was reported that successful star formation could exclude PBH dark matter with $10^{16}\mathrm{g} < M_\mathrm{PBH} < 10^{26}\mathrm{g}$ [27].

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There are another types of observation by which we can check the large running. If $\mathcal{P}_\zeta$ is larger than $\gtrsim 10^{-6}$ at small scales, Ultra compact Mini Halos (UCMHs) can be produced, which may be checked by observing future lensing events or gamma-ray due to dark matter annihilation [30, 31]. The observation may help distinguish the running-mass or the Type-III Hilltop inflation model from the inflating curvaton model, since in the running-mass inflation model the perturbation on the CMB scale is fixed by the usual CMB observation and therefore its tail could be observed in smaller scales, while there is no such bound in the inflating curvaton model.

A. Model-dependence

Basically, the evolution of the perturbation before the curvaton mechanism is highly mode-dependent. In order to avoid the model-dependence, we considered the ratio $\delta\sigma/\sigma$, for which the model-dependent evolution cancels when the quadratic assumption is valid. Any deviation from the quadratic assumption can lead to a highly mode-dependent result, which does not meet the purpose of this paper.

In our calculation the ratio $\delta\sigma/\sigma$ evolves like constant before the beginning of the curvaton mechanism. Our observation is very simple; since σ and $\delta\sigma$ are sharing the same equation of motion (when the potential is qudratic and the perturbation is beyond the horizon), and the equation is a linear differential equation, their ratio behaves like constant. The typical evolution is shown in Eq.(7) for constant m_σ and H, which gives the identical evolution factor $\sim e^{-\beta\Delta N}$ for both $\delta\sigma_k$ and σ. The evolution is thus cancelled in the ratio.

B. non-Gaussianity

There have been many papers suggesting that the local-type non-Gaussianity could prevent PBH formation [33] when $f_{NL} < -0.5$. This bound could be crucial for the oscillation curvaton, since in that model one may find $f_{NL} \simeq -1$ when $r_\sigma \simeq 1$. On the other hand, the exact non-Gaussianity in the inflating curvaton scenario has been calculated in Ref.[12], which shows

$$
\begin{aligned}
f_{NL} =\ & \frac{1}{r_\sigma}\frac{5\epsilon_w}{4R}\left(\frac{gg''}{g'^2}-1\right)\\
& -5\epsilon_w + \frac{10}{3} + \left(\frac{5}{2}\epsilon_w - \frac{10}{3}\right)r_\sigma\\
& +\frac{5}{2}\epsilon_w\left(\frac{1}{r_\sigma}-\frac{1}{R}\right),
\end{aligned}
\tag{26}
$$

where the ratio is defined as $R \equiv \pm\frac{1}{2}m_\sigma^2\sigma^2/V_0$. Note that R is negative for the hilltop potential. With the quadratic assumption we have

$$
\begin{aligned}
g' &= \frac{dg}{d\sigma_*} = \frac{g}{\sigma_*}\\
g'' &= \frac{-g+g'\sigma_*}{\sigma_*^2} = 0.
\end{aligned}
\tag{27}
$$

Here the non-Gaussianity is evaluated for the PBH scale perturbation of the inflating curvaton.

Unlike the oscillating curvaton, $r_\sigma \simeq 1$ in the inflating curvaton does not lead to $f_{NL} \sim -1$. For $r_\sigma \simeq 1$ there are cancellations in Eq.(26), which finally gives $f_{NL} > 0$ for $R < 0$. For $r_\sigma < 1$ there is no cancellation but Eq.(26) gives $f_{NL} > 0$.

We thus find that the sign of the non-Gaussianity parameter is plausibly positive in our model. The problem of the oscillating curvaton is avoided in the inflating curvaton.

III. CONCLUSION AND DISCUSSION

In this paper we have shown that the PBH formation with the typical mass range $10^{14}\text{g} \leq M_{\text{PBH}} \leq 10^{38}\text{g}$ is possible if the curvaton potential is flipped due to the Hubble-induced mass.

We have been avoiding highly model-dependent argument, but the result suggests that the inflating curvaton can generate the PBH that could be the dark matter, and/or the PBH with the interesting scale that could be observable in future observations [17, 18].

The secondary inflation stage, which has been considered in this paper, sometimes plays important role in specific cosmological models. Although it depends on the inflationary model, the spectral index can be tuned by changing N_1, which usually leads to enhanced running of the spectral index. In other cases one may expect fast (rapid) rolling inflaton for the primordial inflation, which requires additional expansion. In both cases the secondary inflation is mandatory for the scenario.

The inflating curvaton can also dilute unwanted cosmological relics that are produced after primordial inflation [32]. In the most optimistic case, the inflating curvaton solves above problems and at the same time it explains the dark matter of the Universe.

Neither artificial interaction nor fine-tuning of the potential has been assumed for the mechanism.

Our result also suggests that the Universe that has many isocurvature components may naturally lead to significant short-length perturbations. Even if they will not source PBH formation, they might leave observable signal in the small scale perturbations.

In addition, at small scales the second order induced gravitational wave signal can be larger than the first order in these types of running models [18]. In Ref.[18], Alabidi et. al. studied this effect and showed that one could discriminate models by observing gravitational wave in future project BBO/DECIGO. Those observations at small scales may complement CMB observations

at large scales, or UCMHs and μ-distortion at small scales to reveal the physics related to the PBH formation.

IV.   ACKNOWLEDGMENT

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